The Fundamental Constants in Physics

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February 17, 2009

Abstract

We discuss the fundamental constants of Physics in the Standard Model and possible changes of these constants on the cosmological time scale. The Grand Unification of the strong, electromagnetic and weak interactions implies relations between the time variation of the finestructure constant α and of the QCD scale Λ_c. A change of α by 10^{-15} / year, as seen by an astrophysics experiment, implies thus a time variation of Λ_c of at least 10^{-15} / year. An experiment in Quantum Optics at the MPQ in Munich, which was designed to look for a time variation of Λ_c, is discussed.

Contents

1 The Standard Model

The Standard Model consists of

a) the gauge theory of the strong interactions: Quantum Chromodynamics (QCD)[1],

b) the gauge theory of the electroweak interactions, based on the gauge group $SU(2) \times U(1)$[2].

QCD is an unbroken gauge theory, based on the gauge group $SU(3)$, acting in the internal space of "color". The basic fermions of the theory are the six quarks, which form color triplets. The gluons, the eight massless gauge bosons, are $SU(3)-$ octets. The interactions of the quarks and gluons are dictated by the gauge properties of the theory. The quarks and gluons interact through the vertex $g_s \cdot \bar{q}\gamma_\mu \frac{\lambda_i}{2} q \cdot A_i^\mu$, where q are the quark fields and A_i^μ the eight gluon fields. The eight $SU(3)$–matrices are denoted by λ_i. The strength of the coupling constant is given by g_s.

QCD is a non–Abelian gauge theory. There is a direct coupling of the gluons among each other. There is also a trilinear coupling, proportional to g_s, and a quadrilinear coupling, proportional to g_s^2. It is assumed, that the QCD interaction leads to a confinement of all colored quanta, in particular of the quarks and the gluons. But this is thus far not proven. Replacing the continuous space–time continuum by a lattice, one can solve the QCD field equations with the computer. The results confirm the confinement hypothesis.

The experimental data are in very good agreement with QCD [3]. Quantum Chromodynamics has the property of asymptotic freedom. The strength of the quark-gluon-interaction converges to zero on a logarithmic scale at high energies. At low energies the interaction strength is large. Thus the confinement property of QCD might indeed be true.

The equations, describing the renormalization of the coupling constant, give for $\alpha_s = \frac{{g_s}^2}{4\pi}$:

$$\begin{aligned} \mu \cdot \frac{\partial \alpha_s}{\partial_\mu} &= -\frac{\beta_0}{2\pi}\alpha_s^2 - \frac{\beta_1}{4\pi^2}\alpha_s^3 - \ldots \\ \beta_0 &= 11 - \frac{2}{3}n_f \\ \beta_1 &= 51 - \frac{19}{3}n_f \end{aligned} \tag{1}$$

(n_f: number of relevant quark flavors)

Since the interaction is weak at high energies, the quarks and gluons appear nearly as pointlike objects at small distances. This has been observed in the experiments of deep inelastic scattering of electrons, myons and neutrinos off nuclear targets.

The strong coupling constant at high energies is small, but not zero. Therefore one expects violations of the scaling behaviour of the cross-sections. This has been seen in many experiments. The value of the QCD coupling constant $\alpha_s = \frac{{g_s}^2}{4\pi}$ depends on the energy. One has found in the analysis of scaling violations[3]:

$$\alpha_s\left(M_z^2\right) \approx 0.1187 \pm 0.002 \tag{2}$$

(M_z: mass of the Z–boson, $M_z \cong 91.2$ GeV).

We can express $\alpha_s(\mu)$ as a function of the scale parameter of QCD Λ_c:

$$\begin{aligned} \alpha_s(\mu)^{-1} &\approx \left(\frac{\beta_0}{4\pi}\right) ln\left(\frac{\mu^2}{\Lambda_c^2}\right) \\ \beta_0 &= \left(11 - \frac{2}{3}n_f\right) \end{aligned} \tag{3}$$

The experiments give the following value:

$$\Lambda_c \approx 217^{+25}_{-23} \text{ MeV}\,. \tag{4}$$

The electroweak gauge theory is based on the gauge group $SU(2) \times U(1)$. Thus there are three W–bosons, related to the $SU(2)$ group, and a B–boson, related to the $U(1)$–group. The lefthanded quarks and leptons are $SU(2)$–doublets, the righthanded leptons and quarks are singlets. Parity is violated in a maximal way.

The gauge invariance of the $SU(2) \times U(1)$–model is broken by the "Higgs"–mechanism[4]. The masses of the gauge bosons are generated by a spontaneous symmetry breaking. Goldstone bosons appear as longitudinal components of the gauge bosons. In the standard "Higgs" mechanism there exists a self–interacting complex doublet of scalar fields. In the process of symmetry breaking the neutral component of the scalar doublet acquires a vacuum expectation value v, which is determined by the Fermi constant of the weak interactions. Therefore the vacuum expectation value is known from the experiments, if the theory is correct:

$$v \cong 246 \ \text{GeV} \tag{5}$$

This energy sets the energy scale for the electroweak symmetry breaking. Three massless Goldstone bosons are generated, but they are absorbed to give masses to the W^+, W^- and Z–bosons. One component of the complex doublet is not absorbed. This is the "Higgs"–boson, thus far a hypothetical particle. It would be the only elementary scalar boson in the Standard Model. One hopes to find this particle with the new accelerator LHC at CERN (start in 2009).

In the electroweak model one has two neutral gauge bosons, which are mixtures of W_3 and B, the Z–boson and the photon. The associated electroweak mixing angle Θ_w is a fundamental parameter which has to be fixed by experiment. It is given by the Z–mass, the Fermi constant and the fine structure constant α:

$$sin^2\Theta_w \cdot cos^2\Theta_w = \frac{\pi\alpha\left(M_z\right)}{\sqrt{2} \cdot G_F \cdot M_Z^2} \,. \tag{6}$$

In the experiments one finds $sin^2\Theta_w \approx 0.231$.

Note that the electroweak mixing angle is also related to the mass ratio M_W/M_Z. If one neglects radiative corrections, one finds:

$$\begin{aligned} sin^2\Theta_w &= 1 - M_W^2/M_Z^2 \\ M_Z &= M_W/cos\Theta_w \,. \end{aligned} \tag{7}$$

In the Standard Model the interactions depend on 28 fundamental constants. These are:
the constant of gravitiy G,
the finestructure constant α,
the coupling constant g_w of the weak interactions,
the coupling constant g_s of the strong interactions,
the mass of the W-boson,
the mass of the "Higgs"–boson,
the masses of the three charged leptons, m_e, m_μ, m_τ,
the neutrino masses $m(\nu_1), m(\nu_2), m(\nu_3)$,
the masses of the six quarks $m_u, m_d, m_c, m_s, m_t, m_b$,
the four parameters, describing the flavor mixing of the quarks,
and the six parameters, describing the flavor mixing of the leptons, measured by the neutrino oscillations.

In physics we are dealing with the laws of nature, but little thought is given to the boundary condition of the universe, related directly to the Big Bang. We do not know at the moment, what role is played by the fundamental costants,

but these constants could form a bridge between the boundary conditions and the local laws of nature. Thus they would be accidental relics of the Big Bang.

Some physicists believe that at least some of the fundamental constants are just cosmic accidents, fixed by the dynamics of the Big Bang. Thus the constants are arbitrary, depending on details of the Big Bang. Obviously in this case there is no way to calculate the fundamental constants.

Some fundamental constants might be cosmic accidents, but it is unlikely, that this is the case for all fundamental constants. New interactions, discovered e. g. with the new LHC–accelerator at CERN, might offer a way to calculate at least some of the fundamental constants.

We also do not understand, why the fundamental constants are constant in time. Small time variations are indeed possible and even suggested by astrophysical experiments. In the theory of superstrings one expects time variations of the fundamental constants, in particular of the finestructure constant, of the QCD scale parameter Λ_c, and of the weak interaction coupling constant[5, 6].

If one finds that the fundamental constants are changing in time, then they are not just numbers, but dynamical quantities which change according to some deeper laws that we have to understand. These laws would be truly fundamental and may even point the way to a unified theory including gravity.

2 Fundamental Constants in the Standard Model

The Standard Model of particle physics is the theory of the observed particle physics phenomena. However it depends on 28 fundamental constants. Within the Standard Model there is no way to calculate these constants.

The most famous fundamental constant is the finestructure constant α, introduced in 1916 by Arnold Sommerfeld:

$$\alpha = \frac{e^2}{\hbar c} \,. \tag{8}$$

In this constant the electromagnetic coupling e enters, as well as the constant of the quantum physics h, and the speed of light c. Sommerfeld realized that α is a dimensionless number, close to the inverse of the prime number 137. The experiments give the following value for α^{-1}: 137,03599911(46)[3].

Werner Heisenberg proposed in 1936 the relation:

$$\alpha = 2^{-4}\, 3^{-3} \pi \,, \tag{9}$$

which gives $\alpha^{-1} = 137,51$. In 1971 Wyler[7] published the following expression for α:

$$\alpha = \frac{9}{8\pi^4} \left(\frac{\pi^5}{2^4 \cdot 5!} \right)^{1/4} \,, \tag{10}$$

which gives $\alpha^{-1} = 137,03608$.

Richard P. Feynman wrote about the finestructure constant[8]: "It has been a mystery ever since it was discussed more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it. Immediately you would like to know where this number for a coupling comes from: is it related to π or perhaps to the base of the natural logarithms? Nobody

knows. It's one of the greatest mysteries of physics: a magic number that comes to us with no understanding by man ...".

In quantum field theory the strength of an interaction is not a fixed constant, but a function of the energy involved. The groundstate of a system is filled with virtual pairs of quanta, e.g. with e^+e^-–pairs in QED. Thus an electron is surrountded by e^+e^-–pairs. The virtual electrons are repelled by the electrons, the virtual positrons are attracted. The electron charge is partially shielded by the virtual positrons. At relatively large distances the electron charge is smaller than at distances less than λ_c. The dependence on the energy is described by the renormalization group equations of Murray Gell–Mann and Francis Low[9]:

$$\frac{d}{d\,ln\,(q/M)} e(q) = \beta(e)\,, \tag{11}$$

where

$$\beta(e) = \frac{e^3}{12\pi^2} + \text{ higher order terms }. \tag{12}$$

In QED one has to include not only virtual e^+e^-–pairs, but also the $\mu^+\mu^-$– and $\tau^+\tau^-$–pairs, as well as the quark–antiquark–pairs. One finds that the finestructure constant α at the mass of the Z–boson should be the inverse of 128, in good agreement with the experimental data taken with the LEP–accelerator[3].

Another fundamental parameter of the Standard Model is the mass of the proton. In QCD the proton mass is a parameter, which can be calculated as a function of the QCD scale parameter Λ_c and of the light quark masses. The QCD scale parameter has been determined many experiments:

$$\Lambda_c = 217 \pm 25 \text{ MeV}\,. \tag{13}$$

(Λ_c is defined in the modified minimal subtraction $\left(\bar{M}S\right)$ scheme for five quark flavors).

The QCD theory gives a very clear picture of the mass generation. In the limit, where the quark masses are neglected, the nucleon mass is the confined field energy of the gluons and quarks. It can be written as:

$$M(Nucleon) = const. \cdot \Lambda_c\,. \tag{14}$$

The *const.* has been calculated using the lattice approach to QCD. It is about 3,9, predicting a nucleon mass in the limit $m_q = 0$ of about 860 MeV. The observed nucleon mass (about 940 MeV) is higher, due to the contributions of the mass terms of the light quarks u, d, s, which in reality are not massless.

The mass of the proton can be decomposed as follows:

$$\begin{aligned} M_p \quad = \quad & \text{const. } \Lambda_c + \qquad (15) \\ & < p \mid m_u \bar{u}u \mid p > + < p \mid m_d \bar{d}d \mid p > + < p \mid m_s \bar{s}s \mid p > + c_{\text{elm}} \cdot \Lambda_c\,. \end{aligned}$$

The last term describes the electromagnetic self–energy. It is proportional to the QCD–scale Λ. Calculations give[10]:

$$c_{\text{elm}} \cdot \Lambda_c \approx 2.0 \text{ MeV}\,. \tag{16}$$

The up–quark mass term contributes about 20 MeV to the proton mass, the d–quark mass term about 19 MeV. Thus the d–contribution to the proton mass

is about as large as the u–contribution, although there are two u–quarks in the proton, and only one d–quark. This is due to the fact that the d–mass is larger than the u–mass.

In chiral perturbation theory the u– and d–masses can be estimated[11]:

$$\begin{aligned} m_u &\approx 3 \pm 1 \ \text{MeV} \\ m_d &\approx 6 \pm 1.5 \ \text{MeV}\,. \end{aligned} \tag{17}$$

These masses are normalized at the scale $\mu = 2$ GeV. Note that quark masses are not the masses of free particles, but of dynamical quantities. They depend on the energy scale μ, relevant for the discussion.

The mass of the strange quark can also be estimated in the chiral perturbation theory[11]. One finds at $\mu = 2$ GeV:

$$m_s \approx 103 \pm 20 \ \text{MeV}\,. \tag{18}$$

The mass of the strange quark is about 20 times larger than the d– mass. Although there are no valence s–quarks in the proton, the $\bar{s}s$–pairs contribute about 35 MeV to the proton mass, i. e. more than the $\bar{u}u$– or $\bar{d}d$–pairs, due to the large ratio m_s/m_d. Heavy quarks, e. g. c–quarks, contribute at most ~ 1 MeV to the nucleon mass[12].

We can decompose the proton mass as follows, leaving out the contribution of the heavy quarks:

$$\begin{array}{lllllllllll} M_p & = & 938 \ \text{MeV} & & & & & & & & \\ & = & (862 & + & 20 & + & 19 & + & 35 & + & 2) \ \text{MeV} \\ & & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow \\ & & QCD & & u-quarks & & d-quarks & & s-quarks & & QED \end{array} \tag{19}$$

The masses of the heavy quarks c and b can be estimated by considering the spectra of the particles, containing c– or b–quarks, e. g. the charm–mesons or the B–mesons. One finds[3]:

$$\begin{aligned} m_c &: 1.15 \ \ldots \ 1.35 \ \text{GeV} \ (\bar{M}S - mass) \\ m_b &: \ 4.1 \ \ldots \ 4.4 \ \text{GeV} \ (\bar{M}S - mass. \end{aligned} \tag{20}$$

The dark corner of the Standard Model is the sector of the fermion masses. There are the six quark masses, three charged fermion masses, three neutrino masses, four flavor mixing parameters of the quarks and six flavor mixing parameters of the leptons (if neutrinos are Majorana particles). These parameters make up 22 of the 28 fundamental constants.

What are the fermion masses? We do not know. They might also be due to a confined field energy, but in this case the quarks and leptons would have to have a finite radius, as in composite models. The masses would be generated by a new interaction. The experiments give a limit on the internal radius of the leptons and quarks, which is of the order of 10^{-17} cm[3].

In the Standard Model the masses of the leptons and quarks are generated spontaneously, like the W and Z–masses. Each fermion couples with a certain strength to the scalar "Higgs"–boson via a Yukawa coupling. A fermion mass is then given by:

$$m(fermion) = g \cdot V\,, \tag{21}$$

where V is the vacuum expectation value of the "Higgs"–field. For the electron this Yukawa coupling constant must be very small, since V is about 246 GeV:

$$g(electron) = 0,00000208\,. \tag{22}$$

Nobody understands, why this coupling constant is so small. The problem of fermion masses remains to be solved. It seems to be the most fundamental problem we are facing at the present time. New experiments at the LHC and at the International Linear Collider (ILC) might clarify the issue.

If one is interested only in stable matter, as e. g. in solid state physics, only seven fundamental constants enter:

$$G, \Lambda, \alpha, m_e, m_u, m_d, m_s\,. \tag{23}$$

The mass of the s–quark has been included, since the $(\bar{s}s)$–pairs contribute to the nucleon mass about 40 MeV. These seven constants describe the atoms and molecules.

It is possible, that there exist relations between the fundamental constants. Relations, which seem to work very well, are the relations between the flavor mixing angles and the quark masses, which where predicted some time ago[13]:

$$\begin{aligned} \Theta_u &= \sqrt{m_u/m_c} \\ \Theta_d &= \sqrt{m_d/m_s}\,. \end{aligned} \tag{24}$$

Similar relations ca be derived for the neutrino masses and the associated mixing angles[14].

These relations are obtained if both for the u–type and for the d–type quarks the following mass matrices are relevant (texture 0 matrices):

$$M = \begin{pmatrix} 0 & A & 0 \\ A^* & C & B \\ 0 & B^* & D \end{pmatrix}\,. \tag{25}$$

It would be interesting to know whether such mass matrices are indeed realized in nature.

3 Does the Finestructure Constant depend on Time?

Recent observations in astrophysics[15] indicate that the finestructure constant α depends on the cosmic time. Billions of years ago it was smaller than today. A group of researchers from Australia, the UK and the USA analysed the spectra of distant quasars, using the Keck telescope in Hawaii. They studied about 150 quasars, some of them about 11 billion lightyears away. The redshifts of these objects varied between 0.5 and 3.5. This corresponds to ages varying between 23% and 87% of the age of our universe.

They studied the spectral lines of iron, nickel, magnesium, zinc and aluminium. It was found that α is not constant:

$$\frac{\Delta\alpha}{\alpha} = (-0.72 \pm 0.18) \cdot 10^{-5}\,. \tag{26}$$

Taking into account the ages of the observed quasars, one concludes that in a linear approximation the absolute magnitude of the relative change of α must be:

$$\left|\frac{d\alpha/dt}{\alpha}\right| \approx 1.2 \cdot 10^{-15}/year\,. \qquad (27)$$

But recent observations of quasar spectra, performed by different groups, seem to rule out a time variation of α at the level given above[16, 17].

The idea that the fundamental constants have a cosmological time dependence, is not new. In the 1930s P. Dirac[18] discussed a time variation of Newtons constant G. Dirac argued that the gravity constant should vary by about a factor of two during the lifetime of the universe. The present limit on the time variation of G is: $\dot{G}/G \leq 10^{-11} year^{-1}$[19]. According to Dirac's hypothesis the time variation of G should be about $10^{10}/year$, in conflict with the quoted limit. In the 1950s L. Landau discussed a possible time variation of the finestructure constant α in connection with the renormalization of the electric charge[20].

French nuclear physicists discovered that about 1.8 billion years ago a natural reactor existed in Gabon, West–Africa, close to the river Oklo. About 2 billion years ago uranium -235 was more abundant than today (about 3,7%). Today it is only 0,72%. The water of the river Oklo served as a moderator for the reactor. The natural reactor operated for about 100 million years.

The isotopes of the rare earths, for example the element Samarium, were produced by the fission of uranium. The observed distribution of the isotopes today is consistent with the calculation, assuming that the isotopes were exposed to a strong neutron flux.

Especially the reaction of Samarium with neutrons is interesting[21]:

$$Sm(149) + n \rightarrow Sm(150) + \gamma\,. \qquad (28)$$

The very large cross–section for this reaction (about 60 . . . 90 kb) is due to a nuclear resonance just above threshold. The energy of this resonance is very small: $E = 0.0973$ eV. The position of this resonance cannot have changed in the past 2 billion years by more than 0.1 eV. Suppose α has changed during this time. The energy of the resonance depends in particular on the strength of the electromagnetic interaction. Nuclear physics calculations give:

$$\frac{\alpha\,(Oklo)) - \alpha(now)}{\alpha(now)} < 10^{-7}\,. \qquad (29)$$

The relative change of α per year must be less than 10^{-16} per year, as estimated by T. Damour and F. Dyson[21]. This conclusion is correct only if no other fundamental parameters changed in the past two billion years. If other parameters, like the strong interaction coupling constant, changed also, the constraint mentioned above does not apply.

The Oklo constraint for α is not consistent with the astrophysical observation for the relative changes of α of order 10^{-15} per year. However, if other parameters also changed in time there will be a rather complicated constraint for a combination of these parameters, but there is no inconsistency.

Recently one has also found a time change of the mass ratio

$$\mu = \frac{M(proton)}{m(electron)}\,. \qquad (30)$$

One observed the light from a pair of quasars, which are 12 billion light years away from the earth[22]. This light was emitted, when the universe was only 1.7 billion years old. The study of the spectra revealed, that the mass ratio μ has changed in time:

$$\frac{\Delta\mu}{\mu} \approx (2 \pm 0.6) \cdot 10^{-5} \,. \tag{31}$$

Taking into account the lifetime of 12 billion years, the change of μ per year would be 10^{-15} / year.

4 Grand Unification

In the Standard Model we have three basic coupling constants. The gauge group of the Standard Model is $SU(3)_c \times SU(2) \times U(1)$.

[Show Quoted Text - 660 lines][Hide Quoted Text] The three gauge interactions are independent of each other.

Since 1974 the idea is discussed that the gauge group of the Standard Model is a subgroup of a larger simple group. The three gauge interactions are embedded in a Grand Unified Theory (GUT). A Grand Unification implies that α_3, α_2 and α_1 are related. They can be expressed in terms of the unified coupling constant α_{un} and the energy scale of the unification Λ_u.

The simplest theory of Grand Unification is based on the gauge group $SU(5)$[23]. The quarks and leptons of one generation can be described by two $SU(5)$–representations. Let us consider the 5–representation of $SU(5)$. After the breakdown of $SU(5)$ to $SU(3) \times SU(2) \times U(1)$ one obtains:

$$\begin{aligned} 5 &\rightarrow (3,1) + (1,2) \\ \bar{5} &\rightarrow (\bar{3},1) + (1,2) \,. \end{aligned} \tag{32}$$

The 5–representation contains a color triplet, which is a singlet under $SU(2)$, and a color singlet ($SU(2)$–doublet):

$$(\bar{5}) = \begin{pmatrix} \bar{d}_r \\ \bar{d}_g \\ \bar{d}_b \\ \nu_e \\ e^- \end{pmatrix} \,. \tag{33}$$

The representation with the next higher dimension is the 10–representation, which is an antisymmetric second–rank tensor. The 10–representation decomposes after as follows:

$$(10) \rightarrow (3,2) + (\bar{3},1) + (1,1) \tag{34}$$

In terms of the lepton and quark fields of the first generation we can write the 10–representation (an antisymmetric 5×5–matrix) as follows:

$$(10) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \bar{u}_b & -\bar{u}_g & -\bar{u}_r & -\bar{d}_r \\ -\bar{u}_b & 0 & \bar{u}_r & \bar{u}_g & -\bar{d}_g \\ \bar{u}_g & -\bar{u}_r & 0 & -\bar{u}_b & -\bar{d}_b \\ u_r & u_g & u_b & 0 & e^+ \\ d_r & d_g & d_b & -e^+ & 0 \,. \end{pmatrix} \,. \tag{35}$$

Combining these two representations, one finds the lepton and quarks of one generation:

$$\bar{5} + 10 \rightarrow (3,2) + 2\,(\bar{3},1) + (1,2) + (1,1)\,. \tag{36}$$

For the first generation we have:

$$\bar{5} + 10 \rightarrow \begin{pmatrix} u \\ d \end{pmatrix}_L + \bar{u}_L + \bar{d}_L + \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L + e^+_L\,. \tag{37}$$

The second and third generation are analogous. The unification based on the gauge gruop $SU(5)$ has a number of interesting features:

1) The electric charge is quantized.

$$trQ = O \rightarrow Q(d) = \frac{1}{3}\,Q\left(e^-\right) \tag{38}$$

2) At some high mass scale Λ_{un} the gauge group of the Standard Model turns into the group $SU(5)$, and there is only one single gauge coupling. The three coupling constants g_3, g_2, g_1 for $SU(3), SU(2)$ and $U(1)$ must be of the same order of magnitude, related to each other by algebraic constants.

The rather different values of the coupling constants g_3, g_2, g_1 at low energies must be due to renormalization effects. This would also explain why the strong interactions are strong and the weak interactions are weak. It is related to the size of the corresponding group.

Apart from normalization constants the three coupling constants g_3, g_2 and g_1, are equal at the unification mass Λ_{un}. Thus the $SU(2) \times U(1)$ mixing angle, given by $tan\Theta_w = \frac{g_1}{g_2}$, is fixed at or above Λ_{un}:

$$sin^2\Theta_w = trT_3^2/trQ^2 = \frac{3}{8}\,. \tag{39}$$

At an energy scale $\mu << \Lambda_{un}$ the parameter $sin^2\Theta$ changes along with the three coupling constants:

$$\begin{aligned} \frac{sin^2\Theta_w}{\alpha} - \frac{1}{\alpha_s} &= \frac{11}{6\pi}\,ln\left(\frac{M}{\mu}\right) \\ \alpha/\alpha_s &= \frac{3}{10}\left(6sin^2\Theta_w - 1\right)\,. \end{aligned} \tag{40}$$

At $\mu = M_z$ the electroweak mixing angle has been measured: $sin^2\Theta_w = 0.2312$. Note that above the unification energy α and α_s are related:

$$\alpha/\alpha_s = 3/8\,. \tag{41}$$

This relation can be checked by experiment. In order to get an agreement between the observed values for g_3, g_2 and g_1 and the values predicted by the $SU(5)$ theory, one can easily see that the unification scale must be very high. Note that

$$\begin{aligned} ln\left(\frac{M}{\mu}\right) &= \frac{6\pi}{11}\left(\frac{sin^2\Theta_w}{\alpha} - \frac{1}{\alpha_s}\right) \\ \mu &= M_Z \\ ln\,(M/M_Z) &\cong 39,9 \\ M &\approx 2\cdot 10^{15}\ \text{GeV}\,. \end{aligned} \tag{42}$$

The precise values of the three coupling constants, determined by the LEP–experiments[3], disagree with the $SU(5)$ prediction. The three coupling constants do not converge to a single coupling constant α_{un}[24]. A convergence takes place, if supersymmetric particles are added above the energy of 1 TeV. Supersymmetry implies that for each fermion a boson is introduced (s–leptons, s-quarks), and for each boson a new fermion is introduced (photino, etc.). These new particles are not observed in the experiments. It is assumed that they have a mass of about 1 TeV.

The new particles contribute to the renormalization of the gauge coupling constants at high energies (about 1 TeV). A convergence of the three coupling constants taken place. Therefore a supersymmetric version of the $SU(5)$–theory is consistent with the experiments[24].

In theories of Grand Unification like the $SU(5)$–theory one has quarks, antiquarks and leptons in one fermion representation. Thus the proton can decay, e. g. $p \to e^+\pi^0$. The lifetime depends on the mass scale for the unification. For $\Lambda_{un} = 5 \cdot 10^{14}$ GeV in the $SU(5)$–theory without supersymmetry one finds 10^{30} years for the proton lifetime. The experimental lower limit is about 10^{33} years.

There is a natural embedding of a group $SU(n)$ into $SO(2n)$, due to the fact that n complex numbers can be represented by $2n$ real numbers. One may consider to use the gauge group $SO(10)$ instead of $SU(5)$. This was discussed in 1975 by P. Minkowski and the author[26]. The fermions of one generation are described by a 16–dimensional spinor representation of $SO(10)$.

Since $SU(5)$ is a subgroup of $SO(10)$, one has the following decomposition:

$$16 \to \bar{5} + 10 + 1\,. \tag{43}$$

The fermions of the $SU(5)$–theory are obtained, plus one additional fermion (per family). This state is an $SU(5)$–singlet and describes a lefthanded antineutrino field. Using the leptons and quarks of the first generation we can write the 16–representation as follows in terms of lefthanded fields:

$$(16) = \left(\begin{array}{cccc|cccc} \bar{\nu}_e & \bar{u}_r & \bar{u}_g & \bar{u}_b & u_r & u_g & u_b & \nu_e \\ e^+ & \bar{d}_r & d_y & \bar{d}_b & d_r & d_g & d_b & e^- \end{array} \right) \tag{44}$$

A feature of the $SO(10)$–theory is that the gauge group for the electroweak interactions is larger than in the $SU(5)$–theory. $SO(10)$ has the subgroup $SO(6) \times SO(4)$. Since $SO(4)$ is isomorphic into $SU(2) \times SU(2)$, one finds:

$$SO(10) \to SU(4) \times SU(2)_L \times SU(2)_R\,. \tag{45}$$

The group $SU(4)$ must contain the color group $SU(3)^c$. The 16–representation of the fermions decomposes under $SU(4)$ into two 4–representations. These contain three quarks and one lepton, e. g. (d_r, d_g, d_b) and e^-. One may interpret the leptons as the fourth color. But the gauge group $SU(4)$ must be broken at high energies (higher than at least 1 TeV):

$$SU(4) \to SU(3) \times U(1)\,. \tag{46}$$

We obtain at low energies the gauge group

$$SU(3)^c \times SU(2)_L \times SU(2)_R \times U(1)\,. \tag{47}$$

But the masses of the gauge bosons for the group $SU(2)_R$ must be much larger than the observed W–bosons, related to the group $SU(2)_L$.

In the $SU(5)$–theory the minimal number of fermions of the Standard Model is included. In the $SO(10)$–theory a new righthanded neutrino is added. This righthanded fermion is interpreted as a heavy Majorana particle. A mass for the lefthanded neutrino is generated by the "see–saw"–mechanism[27]. Thus in the $SO(10)$–theory the neutrinos are massive, while in the $SU(5)$–theory they must be massless. The $SO(10)$–theory is more symmetrical than the $SU(5)$–theory. It is hard to believe that Nature would stop at $SU(5)$, if Nature has chosen to unify the basic interactions.

In the $SO(10)$–theory there is one additional free parameter, related to the masses of the righthanded W–bosons. Since righthanded charged currents are not observed, the masses of the associated W–bosons must be rather high, at least 300 GeV[**?**]. There is a new parameter M_R in the $SO(10)$–theory. It can be chosen such that the coupling constant converges at very high energies, without using supersymmetry. If one chooses $M_R \sim 10^9 \ldots 10^{11}$ GeV, the convergence occurs.

The idea of Grand Unification leads to the reduction of the fundamental constants by one. The three gauge coupling constants of the Standard Model can be expressed in terms of a unified coupling constant α_u at the energy Λ_u, where the unification takes place. The three coupling constants $\alpha_s, \alpha_2, \alpha_1$ are replaced by α_u and Λ_u.

In a Grand Unified Theory the three coupling constants of the Standard Model are related to each other. If e. g. the finestructure constant shows a time variation, the other two coupling constants should also vary in time. Otherwise the unification would not be universal in time. Knowing the time variation of α, one should be able to calculate the time variation of the other coupling constants.

We shall investigate here only the time change of the QCD coupling constant α_s.

We use the supersymmetric $SU(5)$–theory to study the time change of the coupling constants[28, 29]. The change of α is traced back to a change of the unified coupling constant at the energy of unification and to a change of the unification energy. These changes are related to each other:

$$\frac{1}{\alpha}\frac{\dot{\alpha}}{\alpha} = \frac{8}{3} \cdot \frac{1}{\alpha_s} \cdot \left(\frac{\dot{\alpha}_s}{\alpha_s} \right) - \frac{10}{\pi} \frac{\dot{\Lambda}_{un}}{\Lambda_{un}}. \tag{48}$$

We consider the following three scenarios:

1) Λ_{un} is kept constant, $\alpha_u = \alpha_u(t)$. We obtain:

$$\frac{1}{\alpha}\frac{\dot{\alpha}}{\alpha} = \frac{8}{3}\frac{1}{\alpha_s}\frac{\dot{\alpha}_s}{\alpha_s}\,. \tag{49}$$

Using the experimental value $\alpha_s\,(M_Z) \approx 0.121$, we find for the time variation of the QCD scale[28]:

$$\begin{aligned} \frac{\dot{\Lambda}}{\Lambda} &\approx R \cdot \frac{\dot{\alpha}}{\alpha} \\ R &\approx 38 \pm 6\,. \end{aligned} \tag{50}$$

The uncertainty in R comes from the uncertainty in the determination of the strong interaction coupling constant α_s. A time variation of the QCD scale Λ implies a time change of the proton mass and of the masses of all atomic nuclei. The change of the nucleon mass during the last 10 billion years amounts to about 0.3 MeV.

In QCD the magnetic moments of the nucleon and of the atomic nuclei are inversely proportional to the QCD scale parameters Λ. We find for the nuclear magnetic moments:

$$\frac{\dot{\mu}}{\mu} = \frac{\frac{d}{dt}\left(\frac{1}{\Lambda}\right)}{\Lambda} = -\frac{\dot{\Lambda}}{\Lambda} = -R \cdot \frac{\dot{\alpha}}{\alpha}\,. \tag{51}$$

Taking the astrophysics result for $(\dot{\alpha}/\alpha)$, we obtain:

$$\frac{\dot{\Lambda}}{\Lambda} \approx 4 \cdot 10^{-14}/yr\,. \tag{52}$$

2) The unified coupling constant is kept invariant, but Λ_{un} changes in time. In that case we find[29]:

$$\frac{\dot{\alpha}}{\alpha} \cong -\alpha \cdot \frac{10}{\pi}\frac{\dot{\Lambda}_{un}}{\Lambda_{un}} \tag{53}$$

and

$$\frac{\dot{\Lambda}}{\Lambda} \approx -31 \cdot \frac{\dot{\alpha}}{\alpha}\,. \tag{54}$$

The change of the unification mass scale Λ_{un} can be estimated, using as input the time variation of the finestructure constant α. Thus Λ_{un} is decreasing at the rate

$$\dot{\Lambda}_{un}/\Lambda_{un} \approx -7 \cdot 10^{-14}/yr\,. \tag{55}$$

The relative changes of Λ and α are opposite in sign. While α, according to ref. [15], is increasing with a rate of $10^{-15}/yr$, the QCD scale Λ and the nucleon mass are decreasing with a rate of about $3 \cdot 10^{-14}/yr$. The magnetic moments of the nucleons and of nuclei would increase:

$$\frac{\dot{\mu}}{\mu} \approx 3 \cdot 10^{-14}/yr\,. \tag{56}$$

3) The third possibility is that both α_u and Λ_{un} are time–dependent. In this case we find:

$$\frac{\dot{\Lambda}}{\Lambda} \cong 46 \cdot \frac{\dot{\alpha}}{\alpha} + 1,07 \cdot \frac{\dot{\Lambda}_{un}}{\Lambda_{un}}\,. \tag{57}$$

On the right two relative time changes appear: $(\dot{\alpha}/\alpha)$ and $\left(\dot{\Lambda}_{un}/\Lambda_{un}\right)$. These two terms might conspire in such a way that $\left(\dot{\Lambda}/\Lambda\right)$ is smaller than about $(\pm 40 \cdot \dot{\alpha}/\alpha)$.

The question arises, whether a time change of the QCD scale parameter could be observed in the experiments. The mass of the proton and the

masses of the atomic nuclei as well as their magnetic moments depend linearly on the QCD scale. If this scale changes, the mass ratio $M_p/m_e = \mu$ would change as well, if the electron mass is taken to be constant.

The mass ratio μ seems to show a time variation – in a linear approximation one has about

$$\frac{\Delta\mu}{\mu} \approx 10^{-15}/year\,. \tag{58}$$

If we take the electron mass to be constant in time, this would imply that the QCD–scale Λ changes with the rate

$$\frac{\Delta\Lambda}{\Lambda} \approx 10^{-15}/year\,. \tag{59}$$

The connection between a time variation of the finestructure constant and of the QCD scale, discussed above, is only valid, if either the unified coupling constant or the unification scale depends on time, not both. If both the unification scale and the unified coupling constant are time dependent, we should use instead eq. (57). There might be a cancellation between the two terms. In this case the time variation of the QCD–scale would be smaller than $10^{-14}/year$. If the two terms cancel exactly, the QCD–scale would be constant, but this seems unlikely. Therefore a time variation of the QCD–scale of the order of $10^{-15}/year$ is quite possible.

Can such a small time variation of Λ_c be observed in the experiments? In Quantum Optics one can carry out very precise experiments with lasers. In the next chapter we shall describe such an experiment at the Max–Planck–Institute of Quantum Optics in mMunich, which was designed especially to find a time variation of the QCD scale Λ_c.

5 Results from Quantum Optics

The hydrogen atom is a very good test object for checking fundamental theories. Its atomic properties can be calculated with very high accuracy. The level structure of the hydrogen atom can be very accurately probed, using spectroscopy methods in the visible, infrared and ultraviolett regions. Thus the hydrogen atom plays an important rôle in determining the fundamental constants like the finestructure constant.

Measurements of the Lamb shift and the 2S hyperfine structure permit very sensitive tests of quantum electrodynamics. Combining optical frequency measurements in hydrogen with results from other atoms, stringent upper limits for a time variation of the finestructure constant[30] and of the QCD scale parameter can be derived.

The employment of frequency combs[31] turned high–precision frequency measurements into a routine procedure. The high accuracy of the frequency comb have opened up wide perspectives for optical atomic clock applications in fundamental physics. Frequency measurements in the laboratory have become competitive recently in terms of sensitivity to a possible time variation of the fine–structure constant. Though the time interval covered by these measurements is restricted to a few years, very high accuracy compensates for this disadvantage. Their sensitivity becomes comparable with astrophysical and geological methods operating on a billion–year time scale.

Important advantages of the laboratory experiments are: The variety of different systems that may be tested, the possibility to change parameters of the experiments in order to control systematic effects, and the determination of the drift rates from the measured data. Modern precision frequency measurements deliver information about the stability of the present values of the fundamental constants, which can only be tested with laboratory measurements. But only non-laboratory methods are sensitive to processes that happened in the early universe, which can be much more severe as compared to the present time.

In the experiment of the MPQ–group in Munich[30] one was able to determine the frequency of the hydrogen 1S–2S–transition to 2466061102474851(34) Hz. A comparison with the experiment performed in 1999 gives an upper limit on a time variation of the transition frequency in the time between the two measurements, 44 months apart. One finds for the difference $(-29 \pm 57) Hz$, i. e. it is consistent with zero.

The hydrogen spectrometer can be interpreted as a clock, like the cesium clock. However in the hydrogen spectrometer one uses a normal transition for the determination of the flow of time. This transition depends on the mass of the electron and on the fine structure constant. In a cesium clock the flow of time is determined by a hyperfine transition, which depends on the fine structure constant, but also on the nuclear magnetic moment.

Comparing the $1S - 2S$ hydrogen transition with the hyperfine transition of Cesium ^{133}Cs, one can obtain information about the time variation of the ratio α/α_s. The Cesium hyperfine transition depends on the magnetic moment of the Cesium nucleus, and the magnetic moment is proportional to $(1/\Lambda_c$, (Λ_c: QCD scale parameter). If Λ_c varies in time, the magnetic moment will also vary.

One has obtained a limit for the time variation of the magnetic moment of the Cesium nucleus[30]:

$$\frac{\delta\mu}{\mu} = (1.5 \pm 2.0s) \cdot 10^{-15}/yr\,. \tag{60}$$

These results are consistent with zero. The limit on the time variation of α is of the same order as the astrophysics result.

The result concerning the magnetic moment implies a limit on the time variation of Λ_c:

$$\frac{\Delta\Lambda_c}{\Lambda_c} = (-1.5 \pm 2.0) \cdot 10^{-15}/yr\,. \tag{61}$$

This result is in disagreement with our results, based on the assumption, that either α_u or Λ_{un} change in time. We obtained about $10^{-14}/yr$, which is excluded by this experiment.

The result given above is consistent with no time change for Λ_c, but it also agrees with a small time change of the order of 10^{-15} per year. If we assume that the electron mass does not change in time, such a change of Λ_c would agree with the astrophysics result on the time variation of the ratio M(proton) / m(electron)[22]. Theoretically we would expect such a time variation, if both Λ_{un} and α_u change in time.

6 Conclusions and Outlook

We have summarized our present knowledge about the fundamental constants and their possible time variation. Today we do not know how these constants are generated or whether they might depend on time. There might be relations between these constants, e. g. between the flavor mixing angles and the fermion masses, or relations between the three coupling constants, implied by the idea of Grand Unification. This would reduce the number of basic constants from 28 down to a smaller number, but at least 18 fundamental constants would still exist.

A possible time variation of the fundamental constants must be rather slow, at least for those fundamental constants, which are measured very precisely, i. e. the finestructure constant, the QCD–scale Λ, and the electron mass. The constant of gravity G is known with a precision of 10^{-11}. All other fundamental constants, e. g. the masses of the other leptons or the masses of the heavy quarks, are not known with a high precision. The present limits on the time variation of the finestructure constant, the QCD scale or the electron mass are of the order of 10^{-15} / year. These limits should be improved by at least two orders of magnitude in the near future.

If the astrophysics experiments indicate a time variation of the order of $10^{-15}/year$, it does not mean that experiments in quantum optics should also give such a time variation. It might be that until about 10 billion years after the Big Bang the constants did vary slowly, but after that they remained constant. No theory exists thus far for a time variation, and there is no reason to believe that a time variation should be linear, i. e. $10^{-15}/year$ throughout the history of our universe. If the fundamental constants do vary, one would expect that the variation is rather large very close to the Big Bang. In the first microseconds after the Big Bang constants like α or Λ_c might have changed by a factor 2, and we would not know.

In cosmology one should consider time variations of fundamental parameters in more detail. Perhaps allowing a suitable time variation of the constants leads to a better understanding of the cosmic evolution immediately after the Big Bang. Allowing time variations might lead to better cosmological theories and to a better understanding of particle physics. Particle physics and cosmology together would give a unified view on the universe.

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